Map Reduce / Workflow Systems II

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Learning Goals Today

- Get to know idea of workflow systems and some examples
- Understand the definition of *communication cost*
- Understand the definition of *wall clock time*
- Get to know theory and intuition of *complexity theory* for MapReduce

Workflow Systems

WORKFLOW SYSTEMS: INTRODUCTION

- Workflow systems generalize MapReduce
- Just as much as MapReduce:
  - They're built on distributed file systems
  - They orchestrate large numbers of tasks with only small input provided by the user
  - They automatically handle failures
- In addition:
  - Single tasks can do other things than just Map or Reduce
  - Tasks interact in more complex ways

Workflow Systems: Flow Graph

- A *function* represents arbitrary functionality within a workflow
  - and not just 'Map' or 'Reduce'
- Functions are represented as *nodes* of the *flow graph*
- Arcs $a \to b$ for two functions a, b mean that the output of function a is provided to function b as input
- *Note:* The same function could be used by many tasks

Workflow Systems

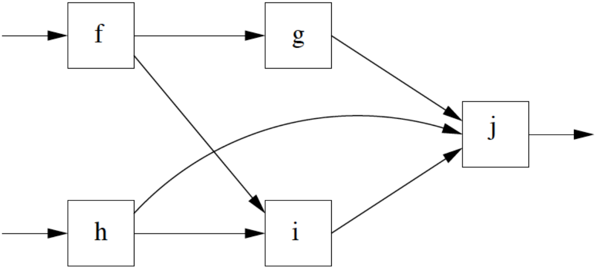


Figure: More complex workflow than MapReduce

Adopted from `mmds.org`

Workflow Systems: Acyclic Flow Graph

- It is easier to deal with *acyclic flow graphs*
  - This means that one cannot return to functions
- *Blocking Property:* tasks only generate output upon completion
  - Blocking property easily applicable only in acyclic workflows
- *Simple Example of Workflow:* Cascades of Map-Reduce jobs
  - Output of Map jobs generated only after all Map tasks are completed
  - Reduce can work only on complete output anyway

Popular Workflow Systems

- *Spark:* developed by UC Berkeley
- *TensorFlow:* Google's system, primarily developed for neural network computations
- *Pregel:* also by Google, for handling *recursive* (i.e. cyclic) workflows
- *Snakemake:* easy-to-use workflow system, inspired by MakeFile logic/functionality

SPARK

- State-of-the-art workflow system:
  - Very efficient with failures
  - Very efficient in grouping tasks among nodes
  - Very efficient in scheduling execution of functions
- Basic concept: *Resilient Distributed Dataset (RDD)*
  - Generalizes key-value pair type of data: RDD is a file of objects of one type
  - *Distributed:* broken into chunks held at different nodes
  - *Resilient:* recoverable from losses of (even all) chunks
- *Transformations* (steps of functions) turn RDD into others
- *Actions* turn other data (from surrounding file system) into RDD's and vice versa

SPARK: TRANSFORMATIONS

Remark: For the following, consider equivalent methods in Python

- *Map* takes a function as parameter and applies it to every element of an RDD, generating a new one
  - Turns one object into exactly another object, but not several ones
  - Remember: Map from MapReduce generates several key-value pairs from one object
- *Flatmap* is like Map from MapReduce, and generalizes it from key-value pairs to general object types (not implemented in Python)
- *Filter* takes a predicate as input
  - Predicate is true or false for elements of RDD
  - So RDD is filtered for objects for which predicate applies
  - Yields a 'filtered RDD'

Spark: Reduce and Relational Database Operations

- *Reduce* is an action, and takes as parameter a function that
  - applies to two elements of a particular type T
  - returns one element of type T
  - and is applied repeatedly until a single element remains
  - Works for associative and commutative operations
- Many *Relational Database Operations* are implemented in Spark:
  - Process RDD's reflecting tuples of relations
  - *Examples:* Join, GroupByKey

Spark: Implementation Details

- Spark is similar like MapReduce in handling data (chunks are called *splits*)
- *Lazy evaluation* allows to apply several transformations consecutively to splits:
  - No intermediate formation of entire RDD's
  - Contradicts blocking property, because partial output is passed on to new functions
- *Resilience* (despite lazy evaluation) is maintained by *lineages of RDD's*
- Beneficial trade-off of more complex recovery of failures versus greater speed overall
  - Note that greater speed reduces probability of failures

TensorFlow

- Open-source system developed (initially) by Google for machine-learning applications
- Programming interface for writing sequences of steps
- Data are *tensors*, which are multidimensional matrices
- Power comes from built-in operations applicable to tensors

RECURSIVE WORKFLOWS

Examples:

- Calculating fixed-points ($Mv = v$ for a matrix M and v) by iterative application of M to v v -> Mv -> M^2v -> M^3v -> … converges to fixed-point
- Gradient descent, e.g. required in TensorFlow for determining optimal sets of parameters for machine learning models
- *Lack of blocking property*:
  - Flow graphs have cycles
  - Tasks may provide their output as input to other tasks whose output in turn results in more input to the first task
  - So generation of output only when task is done does not work
  - *Recovery from failures needs to be reorganized*

Recursive Workflows: Example

- Directed graph stored as relation $E(X, Y)$, listing arcs from X to Y
- Want to compute relation $P(X, Y)$, listing paths from X to Y
- P is transitive closure of E (see below)
- *Algorithm:*
  - *Start:* $P(X, Y) = E(X, Y)$
  - *Iteration:* Add to P tuples

Natural Join: takes (x,z) and (z,y) and generates (x,z,y) for all possible z, so result are possibly several tuples (x,z1,y), (x,z2,y)

Project: both (x,z1,y), (x,z2,y) get (x,y)

$$\pi_{X,Y}(P(X, Z) \bowtie P(Z, Y)) \quad (1)$$

as pairs of nodes X and Y s.t. for some node Z there is path from X to Z and from Z to Y

Transitive Closure: Definition

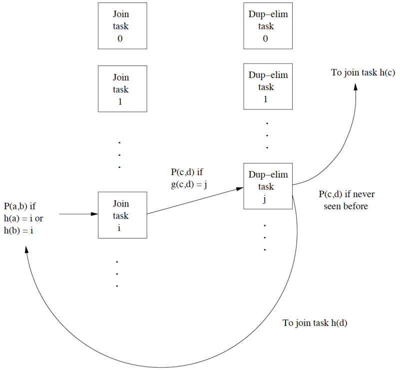
Definition [Transitive Closure]:
Let $R(X, Y)$ be a relation.

- $R(X, Y)$ is *transitive* if $(x, z) \in R$ and $(z, y) \in R$ imply that $(x, y) \in R$ as well
- The *transitive closure* $\overline{R(X, Y)}$ of $R(X, Y)$ is the *smallest set of tuples to be added* to $R(X, Y)$ that renders the resulting set of tuples transitive

Example: Transitive Closure

P(a,b) corresponds to (a,b)

- n Join tasks, corresponding to buckets of hash function h
- Tuple $P(a,b)$ is assigned to Join tasks $h(a)$ and $h(b)$
- i-th Join tasks receives $P(a,b)$
  - Store $P(a,b)$ locally
  - If $h(a) = i$ look for tuples $P(x,a)$ and produce $P(x,b)$
  - If $h(b) = i$ look for tuples $P(b,y)$ and produce $P(a,y)$



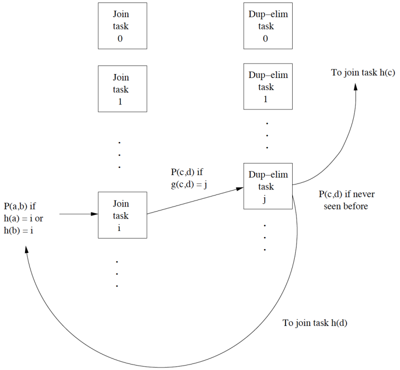
Transitive closure by recursive tasks

locally stored at Join task i: (a,b) and (x,a) => generate (x,b)
locally stored at Join task i: (a,b) and (b,y) => generate (a,y)

Adopted from `mmds.org`

Recursive Workflows: Example

- m Dup-elim tasks, corresponding to buckets of hash function g
- $P(c, d)$ (as output of Join task) is sent to Dup-elim task $j = g(c, d)$
- Dup-elim task j checks whether $P(c, d)$ was received before
  - If *yes*, $P(c, d)$ is ignored (and not stored)
  - If *not*, $P(c, d)$ is stored locally,
  - *and* sent to Join tasks $h(c)$ and $h(d)$



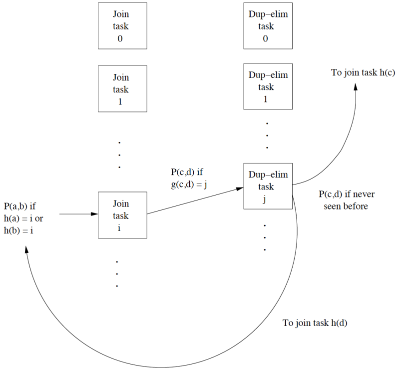
Transitive closure by recursive tasks

Adopted from `mmds.org`

Recursive Workflows: Example

- Every Join task has m output files
- Every Dup-elim task has n output files
- Initially, tuples $E(a,b)$ are sent to Dup-elim tasks $g(a,b)$

E(a,b) is just (a,b)



Transitive closure by recursive tasks

Adopted from `mmds.org`

Recursive Workflows: Failure Handling

- *Iterated MapReduce:* Application is repeated execution / sequence of MapReduce job(s) (“HaLoop”)
- *Spark Approach:* Lazy evaluation, lineage mechanisms, option to store intermediate results
- *Bulk Synchronous Systems:* Graph-based model using “periodic checkpointing”

Bulk Synchronous Systems: Pregel

- System views data as *graph*:
  - *Nodes* (roughly) reflect tasks
  - *Arcs:* from nodes whose output (messages) are input to other nodes
- *Supersteps:*
  - All messages received by any of the nodes from the previous superstep are processed
  - All messages generated are sent to their destinations
- *Advantage:* Sending messages means communication costs, bundling them reduces costs
- *Failure Management:* Checkpointing entire computation by making copy after each superstep
- May be beneficial to checkpoint periodically after number of supersteps

Snakemake

- Create *reproducible* and *scalable* data analyses
- Workflows described in human readable, Python based language
- Seamlessly scale to server, cluster, grid and cloud environments
- Integrating descriptions of required software, deployable to any execution environment

The Communication-Cost Model

Communication Cost

Situation

- Algorithm implemented by acyclic network of tasks:
  - Map tasks feeding Reduce tasks
  - Cascade of several MapReduce jobs
  - More general workflow structure (e.g. Fig. 1)

Definition [Communication Cost]:

- The *communication cost of a task* is the size of the input it receives
- The *communication cost of an algorithm* is the sum of the communication costs of its tasks

Communication Cost

Why Communication Cost?

- Computing communication cost is the way to measure the complexity of distributed algorithm
- Neglect time necessary for tasks to execute
- Importance of communication cost:
  - Tasks tend to be simple (often linear in size of input)
  - Interconnect speed of compute cluster (typically 1 Gbit/sec) slow compared with speed processors execute instructions
  - Often there is competition for the interconnect when several nodes are communicating
  - Moving data from disk to memory may exceed runtime

Why not Output Size?

- Output often is input to another task anyway
- Output rarely large in comparison with input or intermediate data

Reminder: Natural Join

Natural Join: $R(A, B) \bowtie S(B, C)$ (a,b) from R and (b,c) from S get (a, b, c) in the new relation

- **Map:** For each tuple $t = (a, b)$ from R, generate key-value pair $(b, (R, a))$. For each tuple (b, c) from S, generate $(b, (S, c))$.
- **Reduce:** After grouping, each key value b has list of values being either of the form (R, a) or (S, c)
  - Construct all pairs of values where first component is like (R, a) and second component is like (S, c), yielding triples $(b, (R, a), (S, c))$
  - Turn triples into triples (a, b, c) being output

Communication Cost: Natural Join Example

Suppose we are joining $R(A, B) \bowtie S(B, C)$ with R, S of sizes r and s.

- *Map:* Chunks of files R, S are input to Map tasks
  ☞ communication cost of Map is $r + s$ (in practice mostly disk to memory)
- *Reduce:* Input to Reduce tasks is all ($r + s$ many) key-value pairs generated by Map tasks
  ☞ communication cost for Reduce is $O(r + s)$
- *Output of Reduce* could be much larger than $O(r + s)$ (up to $O(rs)$), depending on how many tuples are to be generated for each key b

Communication Cost Example: $R(A, B) \bowtie S(B, C)$

Let sizes of relations R and S be r and s.

Map

- Each chunk of the files holding R and S is fed to one task
  ☞ Communication cost is $r + s$
- Nodes hold chunks already from file distribution step: no internode communication, only disk-to-memory costs
- All Map tasks perform a simple transformation, so only negligible computation cost
- Output about as large as input

COMMUNICATION COST EXAMPLE: $R(A, B) \bowtie S(B, C)$

Let sizes of relations R and S be r and s.

Reduce

- Receives and divides input into tuples from R and S
- For each key, pairs each tuple from R with the ones from S
- Output size can vary: can be larger or smaller than $O(r + s)$
  - Many different B-values: output is small
  - Few B-values: output much larger
- Output large: computation cost could be much larger than $O(r + s)$
- Often output is further subsequently aggregated at further nodes
  ☞ Communication cost greater than computation cost

Wall-Clock Time

Definition [Wall-Clock Time]:

The *wall-clock time* is defined to be the time for the entire parallel algorithm to finish.

Example: Careless reasoning could make one assign all tasks to one node, which minimizes communication cost. But the wall-clock time is (likely to be) at its maximum.

Example: Multiway Join

Consider computing $R(A, B) \bowtie S(B, C) \bowtie T(C, D)$. For simplicity, let us assume that

- the probability that an R- and and S-tuple agree on B
- the probability that an S- and a T-tuple agree on C

are equal. Let p be that probability.

Joining R and S first:

- Communication cost is $O(r + s)$ (see before)
- Size of output is prs
- Hence joining $R \bowtie S$ with T is $O((r + s) + (t + prs))$

Joining S and T first:

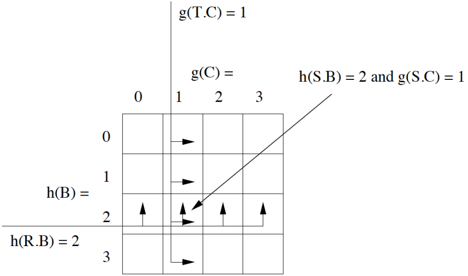
- yields $O((s + t) + (r + pst))$ by analogous considerations

$R(A,B) \bowtie S(B,C) \bowtie T(C,D)$ in one MapReduce

Let p be the probability that an R- and an S-tuple agree on B, matching the probability for an S- and a T-tuple to agree on C.

- Hash B- and C-values, using functions h and g
- Let b and c be the number of buckets for h and g
- Let k be the number of Reducers; require that $bc = k$
  - Each reducer corresponds to a pair of buckets
  - Reducer corresponding to bucket pair (i, j) joins tuples $R(u, v), S(v, w), T(w, x)$ whenever $h(v) = i, g(w) = j$
- Hence Map tasks send R- and T-tuples to more than one reducer
  - R-tuples $R(u, v)$ go to all reducers $(h(v), y)$
    ☞ goes to c reducers
  - T-tuples $T(w, x)$ go to all reducers $(z, g(w))$
    ☞ goes to b reducers

Multiway Join: One MapReduce II



Sixteen reducers for a 3-way join

Adopted from `mmds.org`

- $h(v) = 2, g(w) = 1$
- S-tuple $S(v, w)$ goes to reducer for key $(2, 1)$
- R-tuple $R(u, v)$ goes to reducers for keys $(2, 0), \ldots, (2, 3)$
- T-tuple $T(w, x)$ goes to reducers for keys $(0, 1), \ldots, (3, 1)$

Multiway Join: One MapReduce III

Communication cost:

- Moving tuples to proper reducers is sum of
  - s to send tuples $S(v, w)$ to $(h(v), g(w))$
  - rc to send tuples $R(u, v)$ to $(h(v), y)$ for each of the c possible $g(w) = y$
  - bt to send tuples $T(w, x)$ to $(z, g(w))$ for each of the b possible $h(b) = z$
- Additional (constant) cost $r + s + t$ to make each tuple input to one of the Map tasks (constant)

Multiway Join: One MapReduce III

Communication cost:

- *Goal:* Select b and c, subject to $bc = k$, to minimize $s + cr + bt$
- Using Lagrangian multiplier λ yields to solve for
  - $r - \lambda b = 0$
  - $t - \lambda c = 0$
- It follows that $rt = \lambda^2 bc$, that is $rt = \lambda^2 k$, yielding further $\lambda = \sqrt{\frac{rt}{k}}$
- So, minimum communication cost at $c = \sqrt{\frac{kt}{r}}$ and $b = \sqrt{\frac{kr}{t}}$
- Substituting into $s + cr + bt$ yields $s + 2\sqrt{krt}$
- Adding $r + s + t$ yields $r + 2s + t + 2\sqrt{krt}$, which is usually dominated by $2\sqrt{krt}$

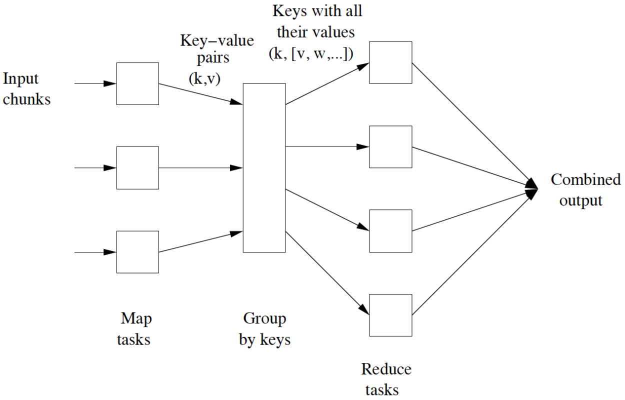
Complexity Theory for MapReduce

MapReduce: Complexity Theory

Idea

- *Reminder:* A "reducer" is the execution of a Reduce task on a single key and the associated value list
- *Important considerations:*
  - Keep communication cost low
  - Keep wall-clock time low
  - Execute each reducer in main memory
- *Intuition:*
  - The less communication, the less parallelism, so
  - the more wall-clock time
  - the more main memory needed
- *Goal:* Develop encompassing complexity theory

REDUCER SIZE: INFORMAL EXPLANATION



Reducer size: maximum length of list [v,w,...] after grouping keys

Adopted from `mmds.org`

REDUCER SIZE

DEFINITION [REDUCER SIZE]:
The *reducer size* q is the upper bound on the number of values to appear in the list of a single key.

Motivation

- Small reducer size forces to have many reducers
- Further creating many Reduce tasks implies high parallelism, hence small wall-clock time
- Sufficiently small reducer size allows to have all data in main memory

Replication Rate

Definition [Replication Rate]:

The *replication rate* r is the number of all key-value pairs generated by Map tasks, divided by the number of inputs.

Motivating Example

- One-pass matrix multiplication algorithm:
  - Matrices involved are $n \times n$
  - *Reminder:* Key-value pairs for MN are $((i,k),(M,j,m_{ij})), j = 1, ..., n$ and $((i,k),(N,j,n_{jk})), j = 1, ..., n$
- Replication rate r is equal to n:
  - Inputs are all m_{ij} and n_{jk}
  - For each m_{ij}, one generates key-value pairs for $(i,k), k = 1, ..., n$
  - For each n_{jk}, one generates key-value pairs for $(i,k), i = 1, ..., n$
- Reducer size is $2n$: for each key (i,k) there are n values from each m_{ij} and n values from each n_{jk}

EXAMPLE: SIMILARITY JOIN

Situation

- Given large set X of elements
- Given similarity measure $s(x, y)$ for measuring similarity between $x, y \in X$
- Measure is symmetric: $s(x, y) = s(y, x)$
- *Output* of the algorithm: all pairs x, y where $s(x, y) \geq t$ for threshold t
- *Exemplary input:* 1 million images (i, P_i) where
  - i is ID of image
  - P_i is picture itself
  - Each picture is 1MB

Example: Similarity Join

MapReduce: Bad Idea

- Use keys (i, j) for pair of pictures $(i, P_i), (j, P_j)$
- *Map*: generates $((i, j), [P_i, P_j])$ as input for
- *Reduce*: computes $s(P_i, P_j)$ and decides whether $s(P_i, P_j) \geq t$
- Reducer size q is small: 2 MB; expected to fit in main memory
- *However*, each picture makes part of 999 999 key-value pairs, so

$$r = 999\,999$$

- Hence, number of bytes communicated from Map to Reduce is

$$10^6 \times 999\,999 \times 10^6 = 10^{18}$$

that is one exabyte

Example: Similarity Join

MapReduce: Better Idea

- Group images into g groups, each of $10^6/g$ pictures
- *Map:* For each (i, P_i) generate $g - 1$ key-value pairs
  - Let u be group of P_i
  - Let v be one of the other groups
  - Keys are sets $\{u, v\}$ (set, so no order!)
  - Value is (i, P_i)
  - Overall: $(\{u, v\}, (i, P_i))$ as key-value pair
- *Reduce:* Consider key $\{u, v\}$
  - Associated value list has $2 \times \frac{10^6}{g}$ values
  - Consider (i, P_i) and (j, P_j) when i, j are from different groups
  - Compute $s(P_i, P_j)$
  - Compute $s(P_i, P_j)$ for P_i, P_j from same group on processing keys $\{u, u + 1\}$

EXAMPLE: SIMILARITY JOIN

MapReduce: Better Idea

- *Replication rate* is $g - 1$
  - Each input element (i, P_i) is turned into $g - 1$ key-value pairs
- *Reducer size* is $2 \times \frac{10^6}{g}$
  - Number of values on list for reducer
  - This yields $2 \times \frac{10^6}{g} \times 10^6$ bytes stored at Reducer node

Example: Similarity Join

MapReduce: Better Idea

- *Example* $g = 1000$*:*
  - Input is 2 GB, fits into main memory
  - Communication cost:

$$\underbrace{(10^3 \times 999)}_{\text{number of reducers}} \times \underbrace{(2 \times 10^3 \times 10^6)}_{\text{reducer size}} \approx 10^{15} \qquad (2)$$

  - 1000 times less than brute-force
  - Half a million reducers: maximum parallelism at Reduce nodes
- *Computation cost* is independent of g
  - Always all-vs-all comparison of pictures
  - Computing $s(P_i, P_j)$ for all i, j

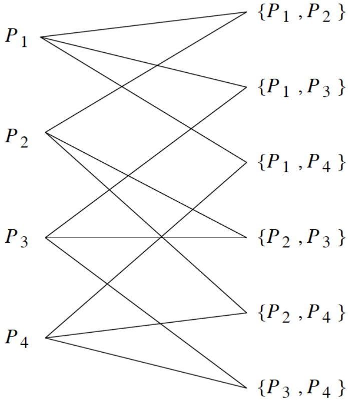
MapReduce: Graph Model

Goal: Proving lower bounds on replication rate as function of reducer size, for many problems. Therefore:

Graph Model:

- Graph describes how outputs depend on inputs
- Reducers operate independently: each output has one reducer that receives all input required to compute output
- *Model foundation:*
  - There is a set of inputs
  - There is a set of outputs
  - Outputs depends on inputs: many-to-many relationship

MapReduce: Graph Model Example



Graph for similarity join with four pictures

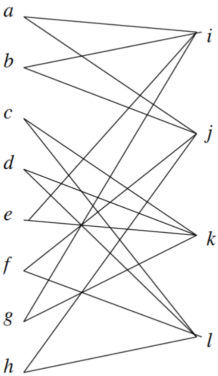
Adopted from `mmds.org`

MapReduce: Graph Model Matrix Multiplication

Graph Model Matrix Multiplication

- Multiplying $n \times n$ matrices M and N makes
  - $2n^2$ inputs $m_{ij}, n_{jk}, 1 \leq i, j, k \leq n$
  - n^2 outputs $p_{ik} := (MN)_{ik}, 1 \leq i, k \leq n$
- Each output p_{ik} needs $2n$ inputs $m_{i1}, m_{i2}, ..., m_{in}$ and $n_{1k}, n_{2k}, ..., n_{nk}$
- Each input relates to n outputs: e.g. m_{ij} to $p_{i1}, p_{i2}, ..., p_{in}$

MapReduce: Graph Model Matrix Multiplication II



$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} i & j \\ k & l \end{bmatrix}$$

Input-output relationship graph for multiplying 2x2 matrices

Adopted from `mmds.org`

MapReduce: Mapping Schemas

A *mapping schema* with a given reducer size q is an assignment of inputs to reducers such that

- No reducer receives more than q inputs
- For every output, there is a reducer that receives all inputs required to generate the output

Consideration: The existence of a mapping schema for a given q distinguishes problems that can be solved in a *single* MapReduce job from those that cannot.

Mapping Schema: Example

Consider computing similarity of p pictures, divided into g groups.

- Number of outputs: $\binom{p}{2} = \frac{p(p-1)}{2} \approx \frac{p^2}{2}$
- Reducer receives $2p/g$ inputs
  ☞ necessary reducer size is $q = 2p/g$
- Replication rate is $r = g - 1 \approx g$:

$$r = 2p/q$$

  ☞ r inversely proportional to q which is common
- In a mapping schema for reducer size q:
  - Each reducer is assigned exactly $2p/g$ inputs
  - In all cases, every output is covered by some reducer

Mapping Schemas: Not all Inputs Present

Example: Natural Join $R(A, B) \bowtie S(B, C)$, where many possible tuples $R(a, b), S(b, c)$ are missing.

- Theoretically $q = |A| \cdot |C|$ (keys were $b \in B$)
- But in practice many tuples $(a, b), (b, c)$ are missing for each b, so q possibly much smaller than $|A| \cdot |C|$

Main Consideration: One can increase q because of the missing inputs, without that inputs do no longer fit into main memory in practice

MAPPING SCHEMAS: LOWER BOUNDS ON REPLICATION RATE

Technique for proving lower bounds on replication rates

1. Prove upper bound $g(q)$ on how many outputs a reducer with q inputs can cover
   ☞ This may be difficult in some cases
2. Determine total number of outputs O
3. Let there be k reducers with $q_i < q, i = 1, ..., k$ inputs
   ☞ observe that $\sum_{i=1}^{k} g(q_i)$ needs to be no less than O
4. Manipulate the inequality $\sum_{i=1}^{k} g(q_i) \geq O$ to get a lower bound on $\sum_{i=1}^{k} q_i$
5. Dividing the lower bound on $\sum_{i=1}^{k} q_i$ by number of inputs is lower bound on replication rate

Lower Bounds: Example All-Pairs Problem

- Recall that $r \leq 2p/q$ was upper bound on replication rate for all-pairs problem
- *Here:* Lower bound on r that is half the upper bound

Lower Bounds: Example All-Pairs Problem

- *Steps from slide before:*
  - Step 1: reducer with q inputs cannot cover more than $\binom{q}{2} \approx q^2/2$ outputs
  - Step 2: overall $\binom{p}{2} \approx p^2/2$ outputs must be covered
  - Step 3: So, the inequality approximately evaluates as

$$\sum_{i=1}^{k} q_i^2/2 \geq p^2/2 \qquad \Longleftrightarrow \qquad \sum_{i=1}^{k} q_i^2 \geq p^2$$

  - Step 4: From $q \geq q_i$, we obtain

$$q \sum_{i=1}^{k} q_i \geq p^2 \qquad \Longleftrightarrow \qquad \sum_{i=1}^{k} q_i \geq \frac{p^2}{q}$$

  - Step 5: Noting that $r = (\sum_{i=1}^{k} q_i)/p$, we obtain

$$r \geq \frac{p}{q}$$

which is half the size of upper bound

MATERIALS / OUTLOOK

- See *Mining of Massive Datasets*, chapter 2.4–2.5
- For deepening your understanding, voluntary *homework*: please read through 2.6.7
- As usual, see `http://www.mmds.org/` in general for further resources
- Next lecture: “MapReduce / Workflow Systems III; Mining Data Streams I”
  - See *Mining of Massive Datasets* 2.6; 4.1–4.7