Lecture 9
Link Analysis II

Alexander Schönhuth

UNIVERSITÄT BIELEFELD
Faculty of Technology

Bielefeld University
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TODAY

Overview

- PageRank Basics Reminder
- PageRank Reality: Structure of the Web
- Topic-Sensitive PageRank: Classify Pages by Topics

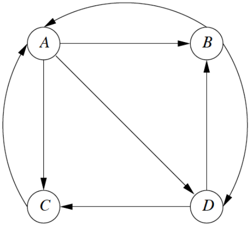
Learning Goals: Understand these topics and get familiarized

PageRank
Reminder

PageRank: Definition

- PageRank is a function that assigns a real number to each (accessible) web page
- *Intuition:* The higher the PageRank, the more important the page
- There is not one fixed algorithm for computing PageRank
- There are many variations, referring to particular issues

PageRank: Definition



Random walking a web with four pages

Adopted from `mmds.org`

- *Random surfer* at B, for example, in next step
  - is at A, D each with probability $1/2$
  - is at B, C with probability 0

Web Transition Matrix: Definition

Definition [Web Transition Matrix]:

- Let n be the number of pages in the web
- The *transition matrix* $M = (m_{ij})_{1 \leq i,j \leq n} \in \mathbb{R}^{n \times n}$ has n rows and columns
- For each $(i, j) \in \{1, ..., n\} \times \{1, ..., n\}$
  - $m_{ij} = 1/k$, if page j has k arcs out, of which one leads to page i
  - $m_{ij} = 0$ otherwise

$$M = \begin{bmatrix} 0 & 1/2 & 1 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix}$$

Transition matrix for web from slides before

Adopted from `mmds.org`

PAGERANK FUNCTION: DEFINITION

DEFINITION [PAGERANK FUNCTION]:

- Let $p_i^t, i = 1, ..., n$ be the probability that the random surfer is at page i after t steps
- The *PageRank function* for $t \geq 0$ is defined to be the vector

$$p^t = (p_1^t, p_2^t, ..., p_n^t) \in [0, 1]^n$$

INSIGHT:

- $p_i^{t+1} = \sum_{j=1}^n m_{ij} p_j^t$ for all i, t
- In other words

$$p^{t+1} = Mp^t \quad \text{for all } t \geq 0 \tag{1}$$

PageRank Function: Markov Processes

$$\boxed{p^{t+1} = Mp^t \quad \text{for all } t \geq 0}$$

- It is known that the surfer reaches a *limiting distribution* $\bar{p}$, characterized by
$$M\bar{p} = \bar{p} \tag{2}$$
- $\bar{p}_i$ is the probability that the surfer is at page i after a long time

Definition [PageRank]:

$\bar{p}_i$ is the *PageRank* of web page i

PageRank Function: Computation

$$\boxed{M\bar{p} = \bar{p}}$$

- For *computing* $\bar{p}$, apply iterative matrix-vector multiplication

$$p^0 \to Mp^0 \to M^2p^0 \to M^3p^0 \to \ldots \tag{3}$$

  until (approximate) convergence
- *Example:* Iterative application of transition matrix from above

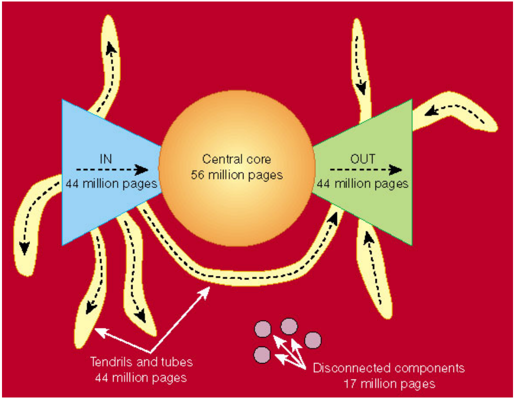
$$\begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix}, \begin{bmatrix} 9/24 \\ 5/24 \\ 5/24 \\ 5/24 \end{bmatrix}, \begin{bmatrix} 15/48 \\ 11/48 \\ 11/48 \\ 11/48 \end{bmatrix}, \begin{bmatrix} 11/32 \\ 7/32 \\ 7/32 \\ 7/32 \end{bmatrix}, \ldots, \begin{bmatrix} 3/9 \\ 2/9 \\ 2/9 \\ 2/9 \end{bmatrix}$$

Convergence to limiting distribution for four-node web graph

Adopted from `mmds.org`

PageRank Reality
Dead Ends and Spider Traps

Structure of the Web



Bowtie picture of the web

Adopted from `mmds.org`

Web Bowtie: Summary

- *Strongly connected component (SCC):* core of the web
- *In-component (IC):*
  - One can reach SCC from IC
  - but not return to IC once left
- *Out-component (OC):*
  - Can be reached from SCC
  - but no longer be left
- *Tendrils:*
  - *First type:* reachable from IC, but can no longer be left
  - *Second type:* can reach OC, but cannot be returned to
- *Tubes:*
  - Can be reached from IC
  - Can only reach OC
- *Isolated components* are not reachable from and cannot reach other components

Bowtie and Markov Chains

Issue: Limiting Distribution

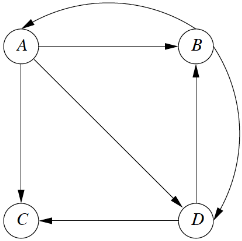
- Random surfers will inevitably wind up in out-component
- Limiting distribution has probability 0 on IC and SCC

☞ No page in IC or SCC of importance

PageRank Modification

- Avoid *dead ends*, single pages with no outlinks
- Avoid *spider traps*, sets of pages without dead ends, but no arcs out
- *Solution: Taxation*
  - Assume random surfer has small probability to leave the web
  - Instead, new surfer starts at random node of the web

Dead Ends



Web graph with dead end (node C)

Adopted from `mmds.org`

- Dead end = columns of all zeroes in the web transition matrix M
- M then is *substochastic* (= column sums at most 1)
- $M^i v$ yields vector with zeroes for certain components
- Dead ends *drain out* the web

Dead Ends

$$M = \begin{bmatrix} 0 & 1/2 & 0 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix}$$

Transition matrix for web with dead end (node C)

Adopted from `mmds.org`

$$\begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix}, \begin{bmatrix} 3/24 \\ 5/24 \\ 5/24 \\ 5/24 \end{bmatrix}, \begin{bmatrix} 5/48 \\ 7/48 \\ 7/48 \\ 7/48 \end{bmatrix}, \begin{bmatrix} 21/288 \\ 31/288 \\ 31/288 \\ 31/288 \end{bmatrix}, \ldots, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Corresponding limiting distribution

Adopted from `mmds.org`

Avoiding Dead Ends

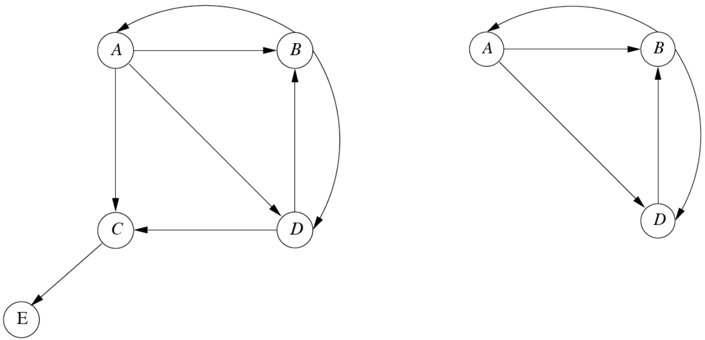
Dropping dead ends: Procedure

- Drop dead ends from graph, and corresponding edges
- Dropping dead ends may create more dead ends
- Keep dropping dead ends iteratively

Dropping dead ends: Consequences

- Removes parts of out-component, tendrils and tubes
- Leaves SCC and in-component

Avoiding Dead Ends



Graph before (left) and after iterative removal of dead ends (right)

Dropping Dead Ends: PageRank Computation

1. After iterative removal of dead ends, compute PageRank for remaining core nodes
2. Re-introduce nodes iteratively, in reverse order relative to their removal
3. PageRank for re-introduced node: sum up PageRank's of predecessors p, divided by the number of successors of p

Dead Ends

$$M = \begin{bmatrix} 0 & 1/2 & 0 \\ 1/2 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

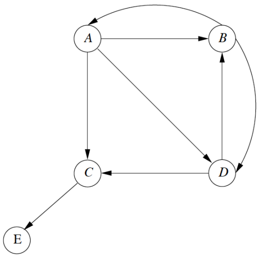
Transition matrix after removal of dead ends

$$\begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}, \begin{bmatrix} 1/6 \\ 3/6 \\ 2/6 \end{bmatrix}, \begin{bmatrix} 3/12 \\ 5/12 \\ 4/12 \end{bmatrix}, \begin{bmatrix} 5/24 \\ 11/24 \\ 8/24 \end{bmatrix}, \ldots, \begin{bmatrix} 2/9 \\ 4/9 \\ 3/9 \end{bmatrix}$$

PageRank(A) = 2/9, PageRank(B) = 4/9, PageRank(D) = 3/9

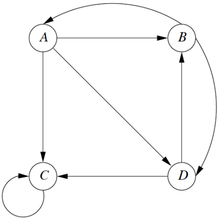
Adopted from `mmds.org`

Dead Ends: PageRank Computation



1. From core: PageRank(A) = 2/9, PageRank(B) = 4/9, PageRank(D) = 3/9
2. Re-introduce node C first:
   $\text{PageRank}(C) = 1/3 \times \text{PageRank}(A) + 1/2 \times \text{PageRank}(D) = \frac{13}{54}$
3. Then re-introduce node E: $\text{PageRank}(E) = 1 \times \text{PageRank}(C) = \frac{13}{54}$

SPIDER TRAPS



Web graph with spider trap (set containing single node C)

Adopted from `mmds.org`

- (Small) group of nodes with no dead ends, but no arcs out
- Can appear intentionally or unintentionally
- "Soak up" all PageRank

SPIDER TRAPS

$$M = \begin{bmatrix} 0 & 1/2 & 0 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 1 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix}$$

Transition matrix for web with single node spider trap (third column)

Adopted from `mmds.org`

$$\begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix}, \begin{bmatrix} 3/24 \\ 5/24 \\ 11/24 \\ 5/24 \end{bmatrix}, \begin{bmatrix} 5/48 \\ 7/48 \\ 29/48 \\ 7/48 \end{bmatrix}, \begin{bmatrix} 21/288 \\ 31/288 \\ 205/288 \\ 31/288 \end{bmatrix}, \dots, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Corresponding limiting distribution

Adopted from `mmds.org`

Spider Traps: Taxation

- Allow the random surfer to get *teleported* to a random page
- *Notation:*
  - Let n be the total number of web pages
  - Let $\mathbf{e} := (1, ..., 1)$ be the vector of length n with all entries one
  - Let β be a small constant; usually $0.8 \leq \beta \leq 0.9$
- *Taxation:* In each matrix-vector multiplication iteration, instead of just computing $\mathbf{v}' = M\mathbf{v}$, compute

$$\mathbf{v}' = \beta M\mathbf{v} + \frac{1}{n}(1-\beta)\mathbf{e} = \beta M\mathbf{v} + (1-\beta)(\frac{1}{n}, ..., \frac{1}{n})^T \tag{4}$$

to obtain a new vector $\mathbf{v}'$ from the actual one $\mathbf{v}$

Spider Traps: Taxation

- *Taxation:* In each matrix-vector multiplication iteration, instead of just computing $\mathbf{v}' = M\mathbf{v}$, compute

$$\mathbf{v}' = \beta M\mathbf{v} + (1-\beta)(\frac{1}{n}, ..., \frac{1}{n})^T$$

  to obtain a new vector $\mathbf{v}'$ from the actual one $\mathbf{v}$
- *Interpretation:*
  - With probability β, the surfer follows an out-link
  - With probability $1-\beta$, the surfer get teleported to a random page
  - In dead ends, surfer disappears with probability β
  - So if there are dead ends, sum of entries in v' less than one
    ☞ So remove dead ends first

Spider Traps

$$\mathbf{v}' = \begin{bmatrix} 0 & 2/5 & 0 & 0 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 0 & 4/5 & 2/5 \\ 4/15 & 2/5 & 0 & 0 \end{bmatrix} \mathbf{v} + \begin{bmatrix} 1/20 \\ 1/20 \\ 1/20 \\ 1/20 \end{bmatrix}$$

Iteration with taxation, with spider trap (third column)

Adopted from `mmds.org`

$$\begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix}, \begin{bmatrix} 9/60 \\ 13/60 \\ 25/60 \\ 13/60 \end{bmatrix}, \begin{bmatrix} 41/300 \\ 53/300 \\ 153/300 \\ 53/300 \end{bmatrix}, \begin{bmatrix} 543/4500 \\ 707/4500 \\ 2543/4500 \\ 707/4500 \end{bmatrix}, \ldots, \begin{bmatrix} 15/148 \\ 19/148 \\ 95/148 \\ 19/148 \end{bmatrix}$$

Corresponding limiting distribution

Adopted from `mmds.org`

PageRank: Efficient Computation

- PageRank virtually is matrix-vector multiplication
  - Consider MapReduce techniques (originally motivated by PageRank)
- *Caveats*, however:
  - Transition matrix M is very sparse; consider appropriate representation of M
  - To reduce communication cost, use combiners
  - Earlier striping technique not sufficient
- So, additional techniques necessary:

see `https://mmds.org`, section 5.2

Topic-Sensitive PageRank

Topic-Sensitive PageRank: Motivation

- Different people have different interests, but ...
- ... different interests are expressed by identical terms
  - E.g. `jaguar` may refer to animal, automobile, operating system, game console
- *Ideally:* Each user has private PageRank vector that measures individual importance of pages
- *But:* It is not feasible to store a vector of length many billions for one billion users

Topic-Sensitive PageRank: Basic Idea

- Identify a (rather small) number of topics
- Compute topic specific PageRank vectors
  - Store topic vectors ...
  - ... instead of individual user vectors
  - There are much less topic vectors
  - *Example for useful topics:* See `https://www.curlie.org/` (new) or `https://www.dmoz-odp.org` for top-level categories
- Assign users to (weighted combination of) topic vectors
- *Drawback:* Looses accuracy
- *Benefit:* Saves massive amounts of space

Topic-Sensitive PageRank: Computation

Idea: Biased Random Walks

- Simulate random surfers that are to prefer pages adhering to particular topics
- Random surfers start at approved topic-specific pages only
- When surfing, they will preferably visit pages linked from topic-specific pages
- Such pages are likely to deal with topic as well
- When being re-introduced (to avoid dead ends, spider traps), surfers again start at approved pages

Topic-Sensitive PageRank: Definition

- Let S be the *teleport set*, i.e. approvedly topic-specific pages
- Let $n, \mathbf{v}, \mathbf{v}', M, \beta$ be as before
- Let $\mathbf{e}_S \in \{0,1\}^n$ be a bit vector of length n such that

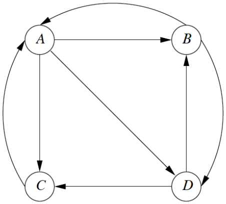
$$\mathbf{e}_S[i] = \begin{cases} 1 & \text{if } i\text{-th page belongs to } S \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

Definition [Topic-Sensitive PageRank]
The *topic-sensitive PageRank for S* is the limit of the iteration

$$\mathbf{v}' = \beta M \mathbf{v} + (1-\beta)\frac{\mathbf{e}_S}{|S|} \tag{6}$$

where $|S|$ is the cardinality (size) of S.

Topic-Sensitive PageRank: Example



Example web graph

Adopted from `mmds.org`

$$\beta M = \begin{bmatrix} 0 & 2/5 & 4/5 & 0 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 2/5 & 0 & 0 \end{bmatrix}$$

Corresponding weighted web transition matrix

Adopted from `mmds.org`

Topic-Sensitive PageRank: Example II

$$\mathbf{v}' = \begin{bmatrix} 0 & 2/5 & 4/5 & 0 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 2/5 & 0 & 0 \end{bmatrix} \mathbf{v} + \begin{bmatrix} 0 \\ 1/10 \\ 0 \\ 1/10 \end{bmatrix}$$

Topic sensitive PageRank computation iteration for teleport set {B,D}

Adopted from `mmds.org`

$$\begin{bmatrix} 0/2 \\ 1/2 \\ 0/2 \\ 1/2 \end{bmatrix}, \begin{bmatrix} 2/10 \\ 3/10 \\ 2/10 \\ 3/10 \end{bmatrix}, \begin{bmatrix} 42/150 \\ 41/150 \\ 26/150 \\ 41/150 \end{bmatrix}, \begin{bmatrix} 62/250 \\ 71/250 \\ 46/250 \\ 71/250 \end{bmatrix}, \ldots, \begin{bmatrix} 54/210 \\ 59/210 \\ 38/210 \\ 59/210 \end{bmatrix}$$

Corresponding limiting distribution

Adopted from `mmds.org`

Topic-Sensitive PageRank: Practical Considerations

- Pick an appropriate set of topics
- For each topic selected, determine teleport set
- *Classifying documents by topic*
  - Has been studied in great detail
  - Topics are characterized by words relating to topic
  - Such words appear surprisingly often in topic-specific pages
  - Determine such words from pages known to relate to topic beforehand
  - Remember the TF.IDF measure (first lecture)

Topic-Sensitive PageRank: Practical Considerations

- When confronted with search query, decide on related topics
- *Determining user-specific topics:*
  - Allow user to choose from menu
  - Infer topics from words appearing in recent queries
  - Infer topics from information on user (bookmarks, stated interests in social media,...)
- Use corresponding topic-sensitive PageRank vectors for ranking responses

Link Spam

Link Spam: Introduction

- Google rendered *term spam ineffective*
- Spammers developed *link spam* as a technique to artificially increase PageRank
- In the following, understand how to
    - create link spam
    - and how to fight it

SPAMMER VIEW OF WEB

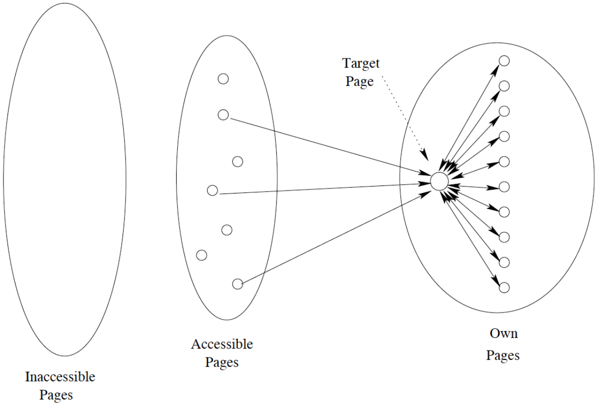
Types of pages

- *Inaccessible pages:* cannot be accessed by spammer; majority of pages
- *Accessible pages:* not owned, but can be accessed (manipulated)
  - ☞ Blogs, newspapers, forums allow leaving comments with links
- *Own pages:* owned and fully controlled by spammer

Spam farm

- Part of own pages with
  - *target page t*, for which maximum PageRank is to be achieved
  - *supporting pages m*, with links from and to *t*
- Note that without links from outside, spam farm would be useless

Spammer View of Web



Spammer view: types of pages and spam farm

Adopted from `mmds.org`

Spam Farm: Analysis

- Let there be n web pages overall
- Let $\beta \in [0.8, 0.9]$ be the taxed fraction of PageRank
- Let there be a spam farm with target page t and m supporting pages
- Let $\mathrm{In}(t)$ be all pages with a link to t; $\mathrm{PR}(p)$ be the PageRank for a page p; $\mathrm{Out}(p)$ be all successors of $p \in P$
- Let

  $$x = \beta \sum_{p \in \mathrm{In}(t)} \frac{\mathrm{PR}(p)}{|\mathrm{Out}(p)|}$$

  be the PageRank provided to t by accessible pages
- Let $y = \mathrm{PR}(t)$ be the unknown PageRank of t
- The PageRank of each supporting page is

  $$\beta \frac{y}{m} + \frac{(1-\beta)}{n}$$

  where $\beta \frac{y}{m}$ is due to t and $\frac{(1-\beta)}{n}$ is due to random teleporting

Spam Farm: Analysis

- Let $y = \mathrm{PR}(t)$ be the unknown PageRank of t
- Let x be the PageRank provided to t by accessible pages
- Let $\beta\frac{y}{m} + \frac{(1-\beta)}{n}$ be the PageRank of each supporting page

Solving for y

1. We compute

$$y = x + \beta m(\frac{\beta y}{m} + \frac{1-\beta}{n}) = x + \beta^2 y + \beta(1-\beta)\frac{m}{n} \tag{7}$$

2. This yields

$$y = \frac{x}{1-\beta^2} + c\frac{m}{n} \tag{8}$$

where $c = \beta(1-\beta)/(1-\beta^2) = \beta/(1+\beta)$

Example: $\beta = 0.85$, so $1/(1-\beta^2) = 3.6$ and $c = 0.46$; spam farm has amplified external contribution to t by 360%; t also obtains 46% of the fraction m/n

Combating Link Spam

War on spam farms

- Search engines identify spam farm structures and eliminate pages from their index
- Spammers create alternative structures that raise PageRank of target pages
- Search engines in turn eliminate those structures, too
- ...
- Endless war between search engines and spammers

Systematic approaches

- *TrustRank:* Variation on topic-sensitive PageRank to lower score of spam pages
- *Spam mass:* Calculation that identifies pages likely to be spam
  ☞ Eliminate such pages or lower their PageRank substantially

TrustRank

- *TrustRank* is like topic-sensitive PageRank where the “topic” are pages believed to be “trustworthy”
  - Inaccessible pages belong to the topic
  - Accessible pages like blogs or newspapers are only borderline trustworthy
- Choosing trustworthy pages:
  1. Human picked pages, or pages of highest PageRank (not achievable by link spam)
  2. Pick pages trustworthy by domain, such as .edu, .ac.uk, .gov and so on

Spam Mass

Definition [Spam Mass]

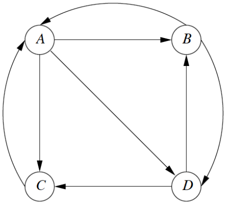
- For a page p, let $r(p)$ and $t(p)$ be its PageRank and its TrustRank
- The *spam mass* of p is defined to be

$$\frac{(r(p) - t(p))}{r(p)}$$

Explanation

- Negative or small spam mass indicates that p is not spam
- Spam mass close to 1 indicates that p is likely to be spam

Spam Mass: Example



Example web graph; B and D are trusted pages

Adopted from `mmds.org`

Node	PageRank	TrustRank	Spam Mass
A	$3/9$	$54/210$	0.229
B	$2/9$	$59/210$	-0.264
C	$2/9$	$38/210$	0.186
D	$2/9$	$59/210$	-0.264

Corresponding page rank, trust rank and spam mass

Adopted from `mmds.org`

MATERIALS / OUTLOOK

- See *Mining of Massive Datasets*, 5.3 – 5.5
- As usual, see `http://www.mmds.org/` in general for further resources
- Next lecture: LinkAnalysis III / Frequent Itemsets I
  - See *Mining of Massive Datasets* chapters 5.5, 6.1, 6.2, 6.3.1, 6.4.1, 6.4.2